

Section 5.3 – Solving Trig Equations

To solve trigonometric equations we use standard algebraic techniques such as collecting like terms and factoring. ALWAYS try to isolate the trig function in the equation.

Examples:

Solve the following trig equations and give all possible solutions in the interval $[0, 2\pi)$.

1. Verifying Solutions of Trig Equations

Decide if the following are solutions to the equation: $\sin 2x - \frac{1}{2} = 0$

a) Is $x = \frac{\pi}{12}$ a solution?

$$= \sin 2\left(\frac{\pi}{12}\right) - \frac{1}{2} = 0 \quad ?$$

$$= \sin \frac{\pi}{6} - \frac{1}{2} = 0 \quad ?$$

$$= \frac{1}{2} - \frac{1}{2} = 0 \quad \checkmark$$

Yes!

b) Is $x = \frac{11\pi}{12}$ a solution?

$$= \sin 2\left(\frac{11\pi}{12}\right) - \frac{1}{2} = 0 \quad ?$$

$$= \sin \frac{11\pi}{6} - \frac{1}{2} = 0 \quad ?$$

$$= -\frac{1}{2} - \frac{1}{2} = -1$$

No!

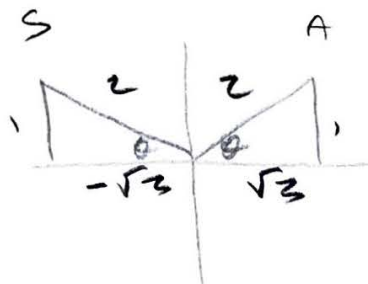
2. Solving by getting the Trig Function alone

$$2\sin\theta = 1$$

$$\frac{2\sin\theta = 1}{2}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$



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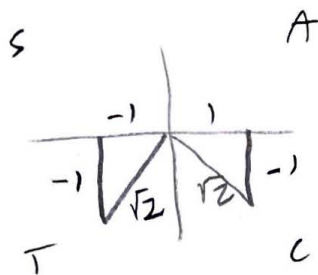
3. Combining Like Terms (onto one side)

$$\sin \theta + \sqrt{2} = -\sin \theta$$

$$2 \sin \theta = -\sqrt{2}$$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$



4. Extracting Square Roots

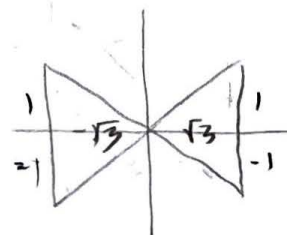
$$3 \tan^2 \theta - 1 = 0$$

$$3 \tan^2 \theta = 1$$

$$\tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$



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5. Factoring

$$\cot \theta \cos^2 \theta = \cot \theta$$

$$\cot \theta \cos^2 \theta - \cot \theta = 0$$

$$\cot \theta (\cos^2 \theta - 1) = 0$$

$$\cot \theta = 0$$

$$\cos^2 \theta = 1$$

$$\cos \theta = \pm 1$$

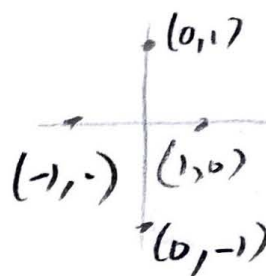
$$\Rightarrow \frac{\cos \theta}{\sin \theta} = 0$$

$$\Rightarrow \cos \theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = 0, \pi$$

Extraneous!!



6. Factoring an Equation of Quadratic Type (may be helpful to use "u" substitution)

$$2\sin^2 \theta - \sin \theta - 1 = 0$$

$$u = \sin \theta$$

$$2u^2 - u - 1 = 0$$

$$(2u+1)(u-1) = 0$$

$$(2\sin \theta + 1)(\sin \theta - 1) = 0$$

$$2\sin \theta + 1 = 0$$

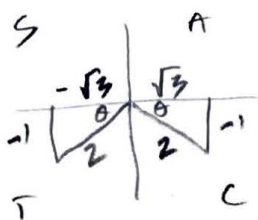
$$\sin \theta - 1 = 0$$

$$\sin \theta = -\frac{1}{2}$$

$$\sin \theta = 1$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\theta = \frac{\pi}{2}$$



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7. Rewriting with a Single Trig Function

$$2\sin^2\theta + 3\cos\theta - 3 = 0$$

$$2(1 - \cos^2\theta) + 3\cos\theta - 3 = 0$$

$$2 - 2\cos^2\theta + 3\cos\theta - 3 = 0$$

$$-2\cos^2\theta + 3\cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta - 1) = 0$$

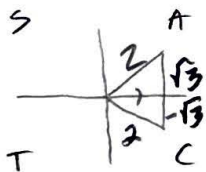
$$\cos\theta = \frac{1}{2}$$

$$\cos\theta - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = 0$$



8. Squaring and Converting to Quadratic Type (check for extraneous roots)

$$\cos\theta + 1 = \sin\theta$$

$$(\cos\theta + 1)^2 = \sin^2\theta$$

$$\cos^2\theta + 2\cos\theta + 1 = 1 - \cos^2\theta$$

$$2\cos^2\theta + 2\cos\theta = 0$$

$$2\cos\theta(\cos\theta + 1) = 0$$

$$2\cos\theta = 0$$

$$\cos\theta = 0$$

$$\cos\theta = -1$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = \pi$$

Extraneous!

$$0 + 1 \neq -1$$

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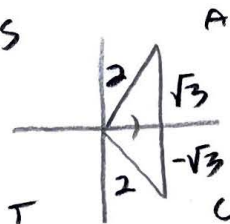
Practice!!

Solve the following equations to find the ANGLE(s) in the interval $[0, 2\pi)$.

1) $\frac{1}{2}\sec x - 1 = 0$

$$\sec x = 2$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$



2) $3\sec^2 x - 4 = 0$

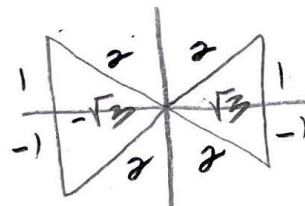
$$3\sec^2 x = 4$$

$$\sec^2 x = \frac{4}{3}$$

$$\sec x = \pm \frac{2}{\sqrt{3}}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$



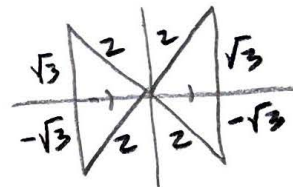
3) $3\cot^2 \theta - 1 = 0$

$$\cot^2 \theta = \frac{1}{3}$$

$$\cot \theta = \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$



4) $3\tan^3 \theta = \tan \theta$

$$3\tan^3 \theta - \tan \theta = 0$$

$$\tan \theta (3\tan^2 \theta - 1) = 0$$

$$\tan \theta = 0 \quad \tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\theta = 0, \pi \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

5) $2\sin^2 x - 3\sin x + 1 = 0$

$$2\sin^2 x - 3\sin x + 1 = 0$$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \quad \sin x = 1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{\pi}{2}$$

6) $4\cos \theta = 1 + 2\cos \theta$

$$2\cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

